PERGAMON

International Journal of Heat and Mass Transfer 42 (1999) 3919–3926

International Journal of
HEAT and MASS
TRANSFER

www.elsevier.com/locate/ijhmt

Effect of small vortex-generators on scalar mixing in the developing region of a turbulent jet

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Received 22 July 1998; received in revised form 5 March 1999

Abstract

This paper investigates the effect of small tabs placed at the exit plane of an axisymmetric smooth contraction on passive scalar mixing in a slightly heated turbulent jet. It is shown that the presence of tabs profoundly distorts the jet flow field and consequently modifies the scalar mixing characteristics significantly. Tabs cause the mean temperature to decrease more rapidly with downstream distance, implying an increased mixing rate. Furthermore, it is found that two tabs distort the jet from the axisymmetric state more dramatically than four tabs. © 1999 Elsevier Science Ltd. All rights reserved.

1. Introduction

The use of small vortex-generating tabs at the nozzle exit is a simple and effective method of enhancing the rate of mixing between a jet and its surroundings and of reducing the noise radiation [1–4]. The impact of tabs on the mean flow of a subsonic round jet appears to have been first observed by Bradbury and Khadem [1]. Recently, the effect of tabs on the near-field evolution of either a supersonic or a subsonic jet has been examined in detail by Zaman et al. [2], Samimy et al. [3], and Reeder and Samimy [4]. Flow visualizations and velocity measurements [2–4] demonstrate that each tab generates one or two pairs of counter-rotating streamwise vortices which dramatically distort the jet cross-section, and that the distortion depends on the number and placement of tabs. These studies have advanced our understanding of the dynamics of the near-field turbulence structure produced by tabs through a comprehensive explanation of the vortex generation process. However, the influence of the perturbation of tabs on the scalar mixing is yet to be investigated.

The statistics of scalar fluctuations in turbulent flows are of both fundamental and practical importance to the understanding of mixing, so that the passive scalar field of a free jet has been extensively investigated for many decades (see e.g. Mi et al. [5] and the references therein). However, our extensive literature search has not found any measurements of scalar quantities, such as passive temperature or mixture fraction, in a turbulent jet from a nozzle with tabs at the exit plane. The present work seeks both to remedy this deficit and to obtain a better understanding of turbulent mixing in the flow within a statistical context. Mi et al. [5] have recently observed significant influence of the initial conditions caused by different types of axisymmetric nozzle (i.e. smooth contraction and pipe) on passive scalar mixing in a turbulent free jet. Even in the far-field self-preserving region, the statistical behavior of a jet flow may be influenced by its initial (exit) conditions. They also deduce that such differences in scalar mixing are associated with differences in the

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0017-9310/99/$ - see front matter
PII: S0017-9310(99)00087-3

Nomenclature

d	diameter of the nozzle exit
f	frequency
f^*	normalised frequency ($\equiv fd/U_o$)
Re_d	Reynolds number ($\equiv U_o d/\nu$)
S_θ, F_θ	skewness ($\equiv \overline{\theta^3}/\overline{\theta^2}^{3/2}$) and flatness ($\equiv \overline{\theta^4}/\overline{\theta^2}^{2}$) of temperature fluctuation θ
T	mean temperature
T_o	mean temperature at the nozzle exit
T_c	mean temperature on the jet centreline

U_o	mean velocity at the nozzle exit
x	downstream distance from the nozzle exit
y, z	lateral distances from the jet centreline

Greek symbols

Φ^*	normalised power spectrum of temperature fluctuation θ, with $\int \Phi^*(f^*)\,df^* = 1$
θ	temperature fluctuation
θ'	r.m.s. temperature fluctuation (($\equiv \overline{\theta^2}^{1/2}$))
ν	kinematic viscosity

underlying structure of the turbulence field. These findings are consistent with the analytic work of George [6] which argues that any turbulent flow cannot become asymptotically independent of its initial conditions. Accordingly, the scalar field of a jet with tabs, which has been unambiguously demonstrated to change significantly the underlying turbulence structure, may be expected to be associated with a different scalar mixing field. That is, the scalar mixing characteristics of the jet flow which is perturbed by tabs at the exit plane may be expected to differ significantly from those without tabs. The present study uses a small temperature differential to act as a passive scalar marker. The temperature field in a slightly heated jet emerging from a smooth contraction nozzle is measured for the conditions with (a) no tabs, (b) two tabs and (c) four tabs.

2. Experimental details

The experiments were carried out in a jet facility in which the flow passes through an electrical heater, a diffuser, a cylindrical plenum chamber and a smoothly contracting nozzle. The nozzle contraction from the 80 mm diameter to the 14 mm exit diameter is described by the relation $R = 40 - 33\sin^{1.5}(90 - 9x'/8)$, see Fig. 1. The nozzle provides a 'top hat' velocity profile (more details of jet initial conditions are given in [5]). Filtered and compressed air was supplied to the facility. The jet exit velocity U_o was controlled by varying the plenum pressure and was measured by a standard pitot tube, while the exit temperature T_o was controlled by varying the power supply to the heater. The diffuser and plenum chamber were vertically placed so that the mean temperature radial profiles within the chamber and at the nozzle exit are approximately symmetrical with respect to the nozzle axis. Some cooling of the air within the nozzle pipe and contraction occurs but the uniformity of temperature distribution at the nozzle exit plane is within ±3.5% based on standard deviation.

Zaman et al. [2] indicate that a triangular-shaped tab (with an apex angle of 90°) leaning downstream at

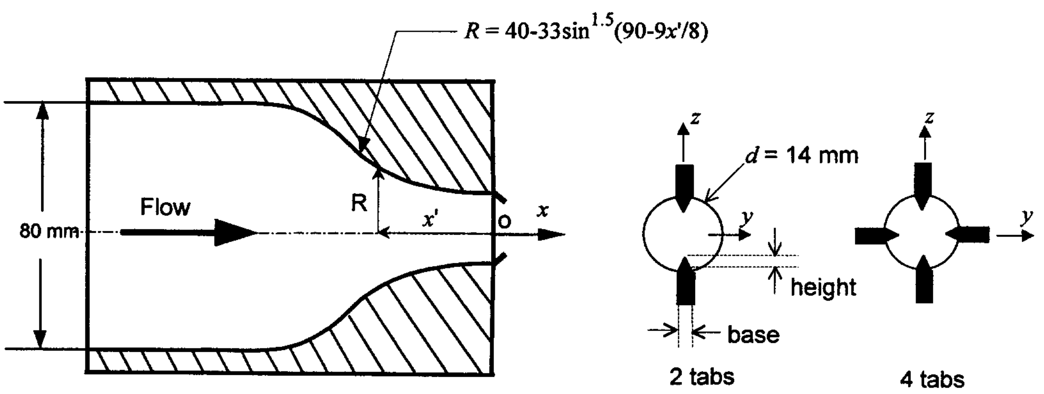

Fig. 1. Sketch of the nozzle and tabs with the coordinate system. Nozzle diameter (d) is 14 mm. Base width and height of the tabs are $0.14d$ and $0.12d$, respectively.

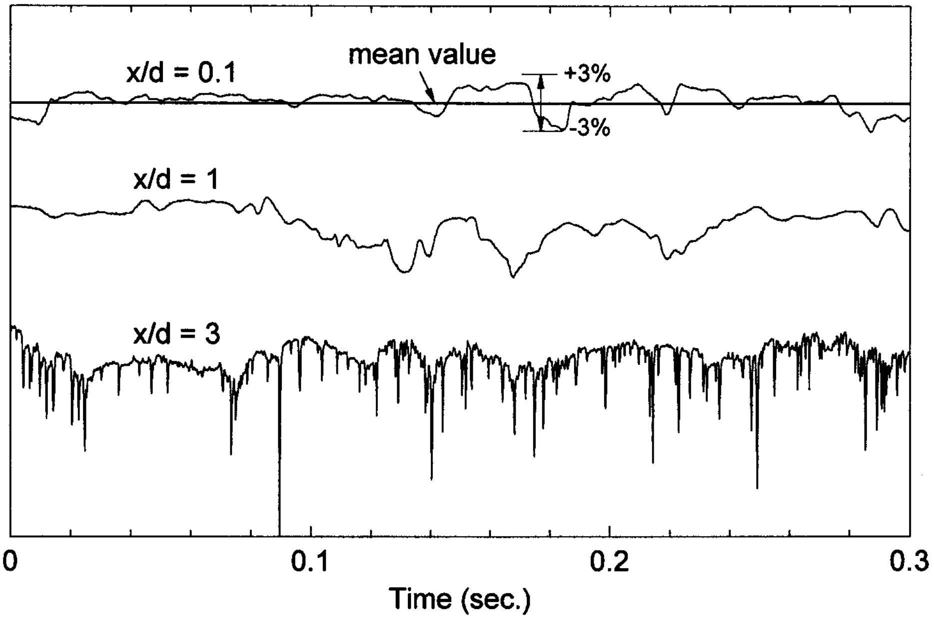

Fig. 2. Centreline temperature signals obtained in the near-field jet without tabs, showing initial non-uniformity and the evolution of the instantaneous temperature in the emerging jet.

45° from the nozzle exit plane produces a greater effect per unit blockage area than a perpendicular rectangular tab. A triangular tab, referred to as the 'delta-tab', with an apex angle of 60° and a leaning angle of 45° (see Fig. 1), was used in the present study. The dimensions shown in Fig. 1 are such that the blockage due to each tab is about 1.2% of the nozzle exit area. The thickness of the boundary layer at the exit from the smooth contraction nozzle is approximately $0.035d$, so that the height of the delta-tabs is therefore nearly four times greater.

Three experiments were performed: one in a jet flow without tabs (a simple jet), one with two tabs and another with four tabs. Fig. 1 shows the arrangement of the nozzle and the tabs. For all cases, the exit Reynolds number Re_d ($\equiv U_o d/\nu$) is approximately 25,000 and the exit temperature T_o is about 51 K above ambient ($T_a \approx 288$ K). These values are chosen to provide a Grashof number Gr_d ($\equiv gd^3 T_o \nu^{-2} T_a^{-1}$) of 12,440 so that the ratio $Gr_d/Re_d^2 \approx 1.2 \times 10^{-5}$ is very small to ensure that buoyancy is negligible and that temperature acts as a passive contaminant.

Temperature measurements in the jets both with two and four tabs were made in the region between $x/d = 0.1$ and $x/d = 23$ in the $y = 0$ and $z = 0$ planes of symmetry. In the jet without tabs, the mean flow is axisymmetric and so measurements were made only in the $y = 0$ plane. A Pt–10%Ph Wollaston wire filament, with diameter of 1.23 μm and length of 0.8 mm, was used as a constant current (0.1 mA) thermometer or referred to as a cold-wire. The output signals were offset, amplified, filtered and then digitised by a personal computer with a 12-bit A/D converter. The signals were filtered at 2.8 kHz to eliminate the high-frequency noise and sampled at a frequency of 5.6 kHz for 20–30 s. Mean temperature (T), r.m.s. ($\theta' \equiv \overline{\theta^2}^{1/2}$), skewness ($S_\theta$), flatness ($F_\theta$) and power spectra ($\Phi(f)$) of temperature fluctuations were calculated afterwards from the temperature signals.

The temperature coefficient of the cold wire ($\approx 1.7 \times 10^{-3}$°C^{-1}) was estimated by calibrating a wire (etched from the same Wollaston spool used for the measurement probe). Experimental errors for various measured quantities have been estimated to be: mean temperature $[T] \approx \pm 2.5\%$; r.m.s. $[\theta'] \approx \pm 2\%$; skewness $[S_\theta] \approx \pm 3\%$; flatness $[F_\theta] \approx \pm 1.5\%$. These estimates were inferred from estimated inaccuracies in the calibration data and from the observed scatter in the measurements.

3. Results and discussion

Fig. 2 shows the near-field, centreline (instantaneous) temperature signals for the jet without tabs. Low-frequency temperature fluctuations (maximum: ±3%) are clearly present in the flow at the jet exit. These fluctuations were deduced to result from nonuniform (instantaneous) temperature distributions within the plenum chamber (since the wall is cooler than the

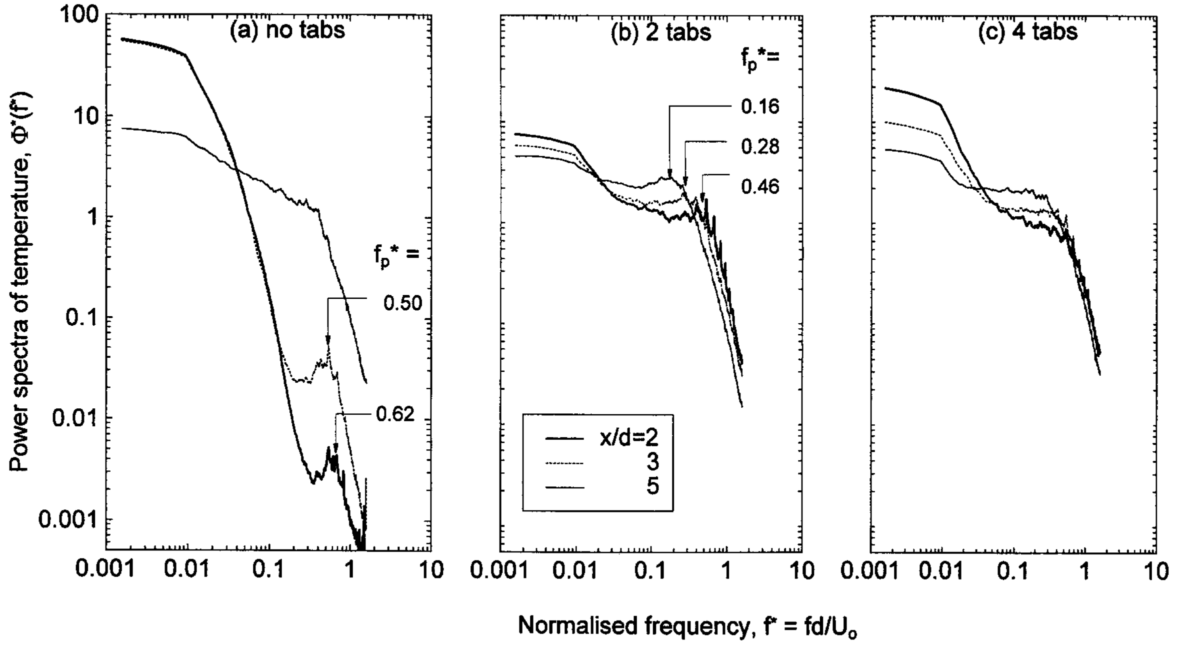

Fig. 3. Normalised power spectra $\Phi^*(f^*)$ showing the influence of tabs on the centreline temperature fluctuations for $x/d = 2$, 3 and 5. Note that $\int \Phi^*(f^*)\,df^* = 1$.

heated air) and the presence of the axisymmetric contraction from 80 to 14 mm diameter. The fluctuations result in relatively high values of the power spectrum at low frequencies (see Fig. 3) and generate quite a high level ($\approx 2.0\% T_o$) of the initial r.m.s. temperature fluctuation θ' (see Figs. 5 and 6 later). However, this value of θ' falls within the error band. Furthermore, the low-frequency fluctuations are present in all the jet flows of interest. Hence, their presence does not influence a comparison of the differences between the scalar-mixing fields in the jets with and without tabs.

Fig. 3(a)–(c) shows the normalised power spectra, $\Phi^*(f^*)$, calculated from the temperature signals θ on the centreline of the three jets with and without tabs at $x/d = 2$, 3 and 5. The normalisation is such that and $\int \Phi^*(f^*)\,df^* = 1$ and $f^* = fd/U_o$. For the jet without tabs, Fig. 3(a), Φ^* clearly has a peak at $f_p^* \approx 0.62$ for $x/d = 2$ and at $f_p^* \approx 0.5$ for $x/d = 3$. These values of f_p^* are consistent with the Strouhal numbers previously identified as being associated with the Kelvin–Helmholtz instability in the initial regions of mixing layers (e.g. [7]). It is generally accepted that, for an axisymmetric jet without tabs, a Kelvin–Helmholtz instability is found in the shear layer produced by a top-hat exit velocity profile and leads to the formation of the ring-like vortex structures in the jet near-field [8]. In the jet with tabs, counter-rotating streamwise vortex pairs are shed from the tabs [2–4] which interact with the larger-scale ring-like vortices and act to break down the original axisymmetry of the flow and produce a more rapid transition to highly three-dimensional turbulence structure. The above deductions of turbulence structure are consistent with Fig. 3. At a given downstream location, the jet with two tabs has a broader and less distinct peak in Φ^* than does the jet without tabs. Likewise, at a given axial distance, the frequency of the peak is lower for the jet with two tabs than the jet without tabs (see Fig. 3(a) and (b)), consistent with a more rapid growth rate of the largest structures and of the width of the jet. With four tabs, the hump is so broad that no clearly defined peak in Φ^* exists. The lack of a distinct peak in this flow with four pairs of streamwise vortices suggests that the interactions of these vortices with each other and with the circumferential ring-like vortices are more complex and three-dimensional than occurs with two tabs so that the flow structures in the jet mixing layer ($x/d \leq 5$) are less coherent in position and/or shape.

Fig. 4(a)–(c) shows lateral distributions of the normalised mean temperature (T/T_o) obtained in the $y = 0$ and $z = 0$ planes of symmetry for each of the three test cases. Clearly the presence of tabs, their distribution and number around the nozzle exit, strongly influences the scalar field of a jet. The presence of two tabs introduces strong asymmetry in the circumferential direction with the jet being substantially broader in the $z = 0$ plane than the $y = 0$ plane throughout the measured region ($x/d \leq 20$). The asymmetry also results in the local maxima of the mean temperature, $T(y, z = 0)$, being located off-axis in the $z = 0$ plane

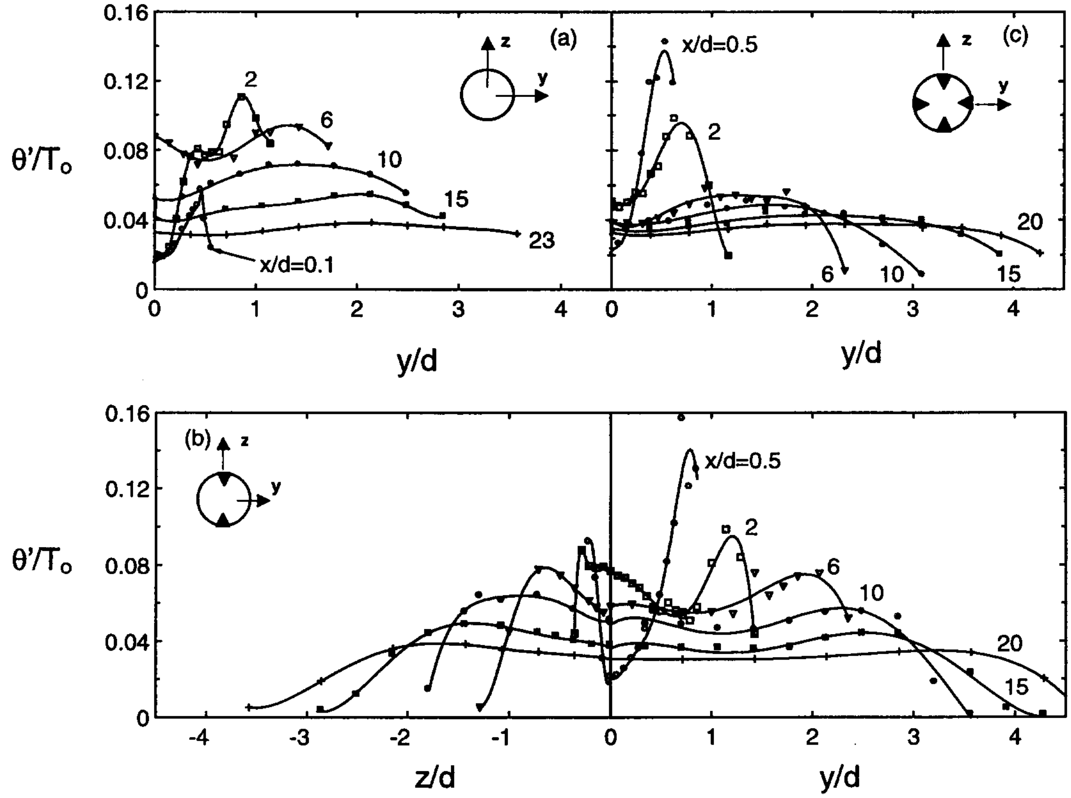

Fig. 4. Lateral mean temperature distributions in the three jet flows: (a) no tabs; (b) two tabs; (c) four tabs. In plot (b), $T(y, z = 0)$ and $T(y = 0, z)$ are presented on the right and left sides, respectively.

for all $x/d < 20$. The local peak in the mean temperature distribution within the $y = 0$ plane, $T(y = 0, z)$, is located on the centreline. The separation between the two peaks in the $z = 0$ plane increases with x/d until $x/d \approx 10$ and then the peaks converge onto the axis at $x/d \approx 20$. While the divergence of peaks has been found in measurements of velocity [1,2], from which it was deduced that the jet core is split into two, the merging of the two 'jet cores' is not indicated in previous works [1–4]. With four symmetrically arranged tabs, $T(y = 0, z)$ vs z is the same as $T(y, z = 0)$ vs y, and so only $T(y, z = 0)$ is plotted in Fig. 4(c). While the mean temperature field of the jet with four tabs is qualitatively similar to that of the unperturbed jet within the measurement plane, it has significantly higher spreading angle and decay rate. It is clear from the above that two tabs distort the jet flow field more strongly than do four tabs, even though the use of four tabs was reported to increase more effectively the rate of jet entrainment of ambient fluid [2].

Lateral distributions of the normalised r.m.s. temperature fluctuation (θ'/T_o) are displayed in Fig. 5(a)–(c) for the three cases. Even without tabs, the distribution of θ'/T_o in the region $x/d < 10$ is relatively complex, probably due to complex flow structures with strong intermittency of the induced non-turbulent ambient air. When $x/d \geq 10$, θ'/T_o reaches local maxima away from the axis. The perturbation of the flow by tabs can be deduced to change the underlying structure of the flow due to the modification of initial conditions from the non-tab case [5]; this is indicated by the different-shape of the distributions of θ'/T_o in the jet near field. The difference is greatest with two tabs, where θ' varies laterally in the $y = 0$ plane in a manner significantly different from that in the $z = 0$ plane, although this difference is eliminated with increasing x. Nevertheless, the lateral distributions in the three cases become more similar as distance from the nozzle increases.

The differences in the three jets are clearly illustrated in Fig. 6 which shows the streamwise variations of T_o/T_c and $(\theta'/T)_c$ along the jet centreline. Several observations can be made from the plot. Firstly, the size of the potential core region is reduced by the perturbation of tabs which demonstrates an increase of the initial rate of mixing by tabs. Note that the region in which $T \approx T_o$ is smaller than (and so does not properly represent) the potential core region, where $U = U_o$, since,

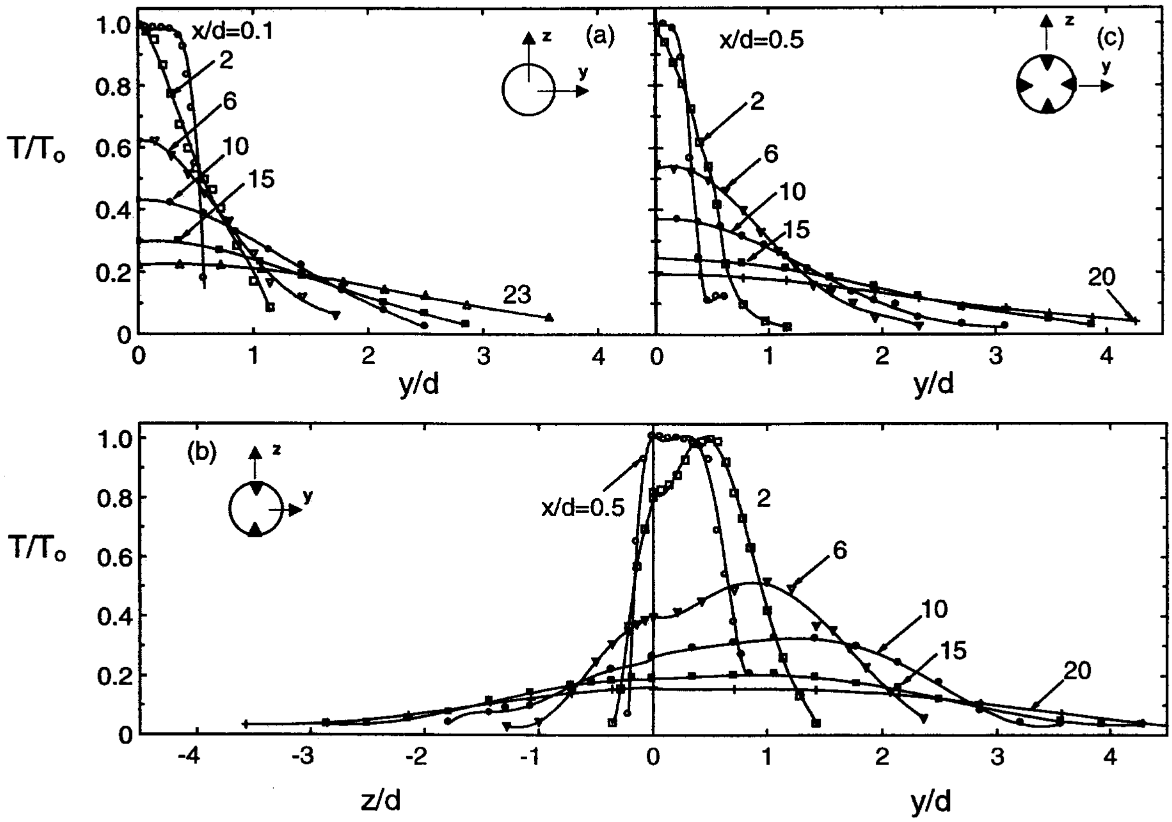

Fig. 5. Lateral distributions of the rms temperature fluctuation in the three jets: (a) no tab; (b) two tabs; (c) four tabs. In plot (b), $\theta'(y, z = 0)$ and $\theta'(y = 0, z)$ are presented by the thick and the thin curves, respectively.

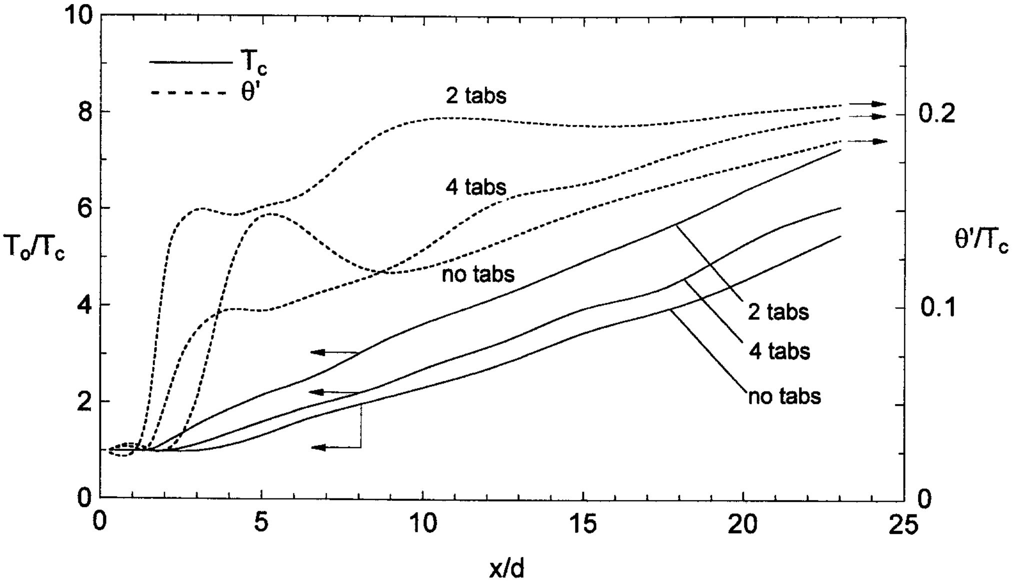

Fig. 6. The normalised distribution of mean (solid lines) and r.m.s. (dashed lines) temperature fluctuations along the centreline.

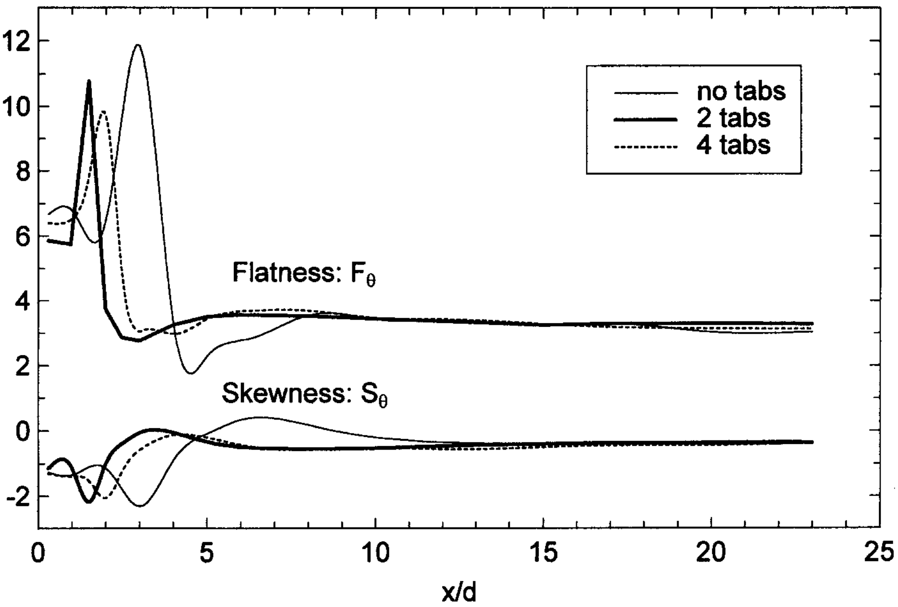

Fig. 7. The centreline variations of the skewness and flatness factors of the temperature fluctuations.

as one reason, the heat diffuses at a higher rate than the momentum in the air flow ($Pr \equiv \nu/\alpha \approx 0.7$); the same observation can be made from the data reported by Chua and Antonia [9] for an axisymmetric free jet. The average length (L_{pc}) of the potential core region in an axisymmetric free jet is about $5d$ based on the mean velocity field [10], and this value is represented by the location (x_{m}) at which $(\theta'/T)_{\mathrm{c}}$ reaches local maximum on the centreline. As indicated in Fig. 6, $x_{\mathrm{m}} \approx 5.4d$ without tabs, $x_{\mathrm{m}} \approx 3d$ with two tabs and $x_{\mathrm{m}} \approx 4d$ with four tabs. Secondly, $T_{\mathrm{o}}/T_{\mathrm{c}}$ varies approximately linearly with x for $x/d > 4$ for all the cases. Clearly, the centreline temperature decay rate $\partial(T_{\mathrm{o}}/T_{\mathrm{c}})/\partial(x/d)$ is highest with two tabs (≈ 0.272) and lowest without tabs (≈ 0.219); in the jet with four tabs, $\partial(T_{\mathrm{o}}/T_{\mathrm{c}})/\partial(x/d) \approx 0.256$. It is interesting to note that Zaman et al. [2] found that four tabs give higher mixing rates than two tabs. The apparent discrepancy may be explained by the fact that the jet with two tabs is split into two cores and the present data are not those on the center of the 'cores'. Furthermore, the ratio of r.m.s. fluctuating to mean scalar $(\theta'/T)_{\mathrm{c}}$ has been shown to asymptote to a constant value in fully developed jet turbulence [5]. For a smooth contraction, the asymptotic value of $(\theta'/T)_{\mathrm{c}}$ is approximately 0.235 which has been reported in both [5] and [11]. It is apparent that the present jet flows have not reached fully developed turbulence in the present range.

The skewness $S_\theta (\equiv \overline{\theta^3}/\overline{\theta^2}^{3/2})$ and flatness $F_\theta \equiv \overline{\theta^4}/\overline{\theta^2}^{2}$) of temperature fluctuation on the centreline are presented in Fig. 7. It is clearly seen that the streamwise variation of either S_θ or F_θ in the near-field region ($x/d \leq 10$) is quite different in the three jets. The axial locations at which F_θ reaches a maximum and S_θ reaches a minimum occurs at $x/d = 3$, 1.4 and 1.8 for zero, two and four tabs, respectively. These differences reflect the influence of the tabs on the flow field and the sensitivity of the influence to the number of tabs. Fig. 7 also shows that, when $x/d \geq 15$, S_θ and F_θ both asymptotically approach their far-field values. Estimated from the present data at $x/d \geq 15$, approximately, those asymptotic values of S_θ and F_θ are −0.32 and 3.08 without tabs; −0.38 and 3.26 with two tabs; and −0.37 and 3.12 with four tabs. Although each pair of these values are not far from the Gaussian counterpart (0, 3), the visible differences still imply that the influence of tabs underlies the flow quite extensively and remains discernible at $x/d = 23$ (and presumably even beyond).

4. Concluding remarks

The passive temperature scalar field was measured in the near-field and intermediate regions ($0.5 \leq x/d \leq 23$) of a turbulent jet emerging from a smoothly contracting axisymmetric nozzle with zero, two or four vortex-generating delta-tabs at the exit plane. The present work has quantified how a change in the initial conditions results in changes in the vortex structures throughout the near and developing region in a manner consistent with the findings of others [2–4]. The perturbation of tabs profoundly distorts the jet flow field and modifies the scalar mixing characteristics. Both lateral and axial distributions of all turbulent quantities of the scalar field are significantly altered. The modification of dynamics of the near-field jet is implied in the power spectrum (Φ) of the temperature fluctuations: the primary peak of Φ associated with the Kelvin–Helmholtz instability of the mixing layer occurs at lower frequencies in the jet with two tabs than without tabs and is a broad hump with no clear peak in the jet with four tabs. In comparison with the unperturbed axisymmetric jet, the mean temperature scalar field in both jets with tabs decays more rapidly with downstream distance, due to an increased rate of mixing between the jet and the induced ambient flow. As a result, the centreline temperature decay rate, $\partial(T_o/T_c)/\partial(x/d)$, is approximately 0.272 with two tabs, 0.256 with four tabs and 0.219 without tabs. The modification of the jet flow field by tabs is also evident in other statistical properties of the passive scalar fluctuations, such as r.m.s., skewness and flatness factors. While the statistical measurements show that differences in the three jets decrease with axial downstream from the nozzle, significant differences are still present at $x/d = 23$. It is deduced from our recent work [5] that these differences are due to different initial conditions and most likely to retain in the far-field self-preserving region.

Scalar measurements including radial mean and r.m.s. temperature profiles, normalised centreline statistics of mean, r.m.s., skewness and flatness all indicate a more profound distortion of the jet cross-section by two tabs than by four tabs. With two tabs, the core of the jet is initially split into two that do not merge until at $x/d \approx 20$.

Acknowledgements

The first author gratefully acknowledges the financial support of the Australian Research Council for this work in 1998. Thanks also go to Dr P.V. Lanspeary for reviewing this paper.

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